

Attribute Grammars

- Pagan: Ch. 2.1, 2.2, 2.3, 3.2
- Stansifer: Ch. 2.2, 2.3
- Slonneger and Kurtz: Ch 3.1, 3.2

Formal Languages

- Basis for the design and implementation of programming languages
- **Alphabet**: finite set Σ of symbols
- **String**: finite sequence of symbols
 - Empty string ε
 - Σ^* - set of all strings over Σ (incl. ε)
 - Σ^+ - set of all non-empty strings over Σ
- **Language**: set of strings $L \subseteq \Sigma^*$

Grammars

- $G = (N, T, S, P)$
 - Finite set of **non-terminal symbols** N
 - Finite set of **terminal symbols** T
 - Starting non-terminal symbol $S \in N$
 - Finite set of **productions** P
- Production: $x \rightarrow y$
 - x is a non-empty sequence of terminals and non-terminals; y is a possibly-empty sequence of terminals and non-terminals
- Applying a production: $uxv \Rightarrow u y v$

Languages and Grammars

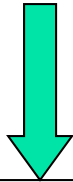
- String derivation
 - $w_1 \Rightarrow w_2 \Rightarrow \dots \Rightarrow w_n$; denoted $w_1 \xRightarrow{*} w_n$
- Language generated by a grammar
 - $L(G) = \{ w \in T^* \mid S \xRightarrow{*} w \}$
- Traditional classification of languages and grammars
 - Regular $\subset$ Context-free $\subset$ Context-sensitive $\subset$ Unrestricted

Regular Languages

- Generated by **regular grammars**
 - All productions are $A \rightarrow wB$ and $A \rightarrow w$
 - $A, B \in N$ and $w \in T^*$
 - Or all productions are $A \rightarrow Bw$ and $A \rightarrow w$
- e.g. $L = \{ a^n b \mid n > 0 \}$ is a regular language
 - $S \rightarrow Ab$ and $A \rightarrow a \mid Aa$
- Alternative equivalent formalisms
 - Regular expressions: e.g. a^*b for $\{ a^n b \mid n \geq 0 \}$
 - Deterministic finite automata (DFA)
 - Nondeterministic finite automata (NFA)

Lexical Analysis in Compilers

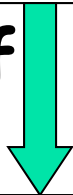
stream of
characters



w,h,i,l,e,(,a,1,5,>,b,b,),d,o,...

Lexical Analyzer (uses a regular grammar)

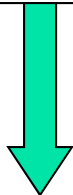
stream of
tokens



keyword[while],leftparen,id[a15],op[>],
id[bb],rightparen,keyword[do], ...

Parser (uses a context-free grammar)

parse
tree



each token is a leaf in the parse tree

... more compiler components

Uses of Regular Languages

- Lexical analysis in compilers
 - e.g., an identifier token is a sequence of characters from the regular language **letter (letter|digit)***
 - each token is a **terminal symbol** for the context-free grammar of the parser
- Pattern matching
 - **stdlinux> grep “a⁺b” foo.txt**
 - Every line from foo.txt that contains a string from the language $L = \{ a^n b \mid n > 0 \}$
 - i.e. the language for reg. expr. a^+b

Context-Free Languages

- Subsume regular languages
 - $L = \{ a^n b^n \mid n > 0 \}$ is c.f. but not regular
- Generated by a **context-free grammar**
 - Each production: $A \rightarrow w$
 - $A \in N, w \in (N \cup T)^*$
- BNF: alternative notation for context-free grammars
 - Backus-Naur form: John Backus and Peter Naur, for ALGOL60 (both have received the ACM Turing Award)

BNF Example

$\langle \text{stmt} \rangle ::= \text{while } \langle \text{exp} \rangle \text{ do } \langle \text{stmt} \rangle$
 $| \text{if } \langle \text{exp} \rangle \text{ then } \langle \text{stmt} \rangle$
 $| \text{if } \langle \text{exp} \rangle \text{ then } \langle \text{stmt} \rangle \text{ else } \langle \text{stmt} \rangle$
 $| \langle \text{exp} \rangle := \langle \text{exp} \rangle$
 $| \langle \text{id} \rangle (\langle \text{exprs} \rangle)$

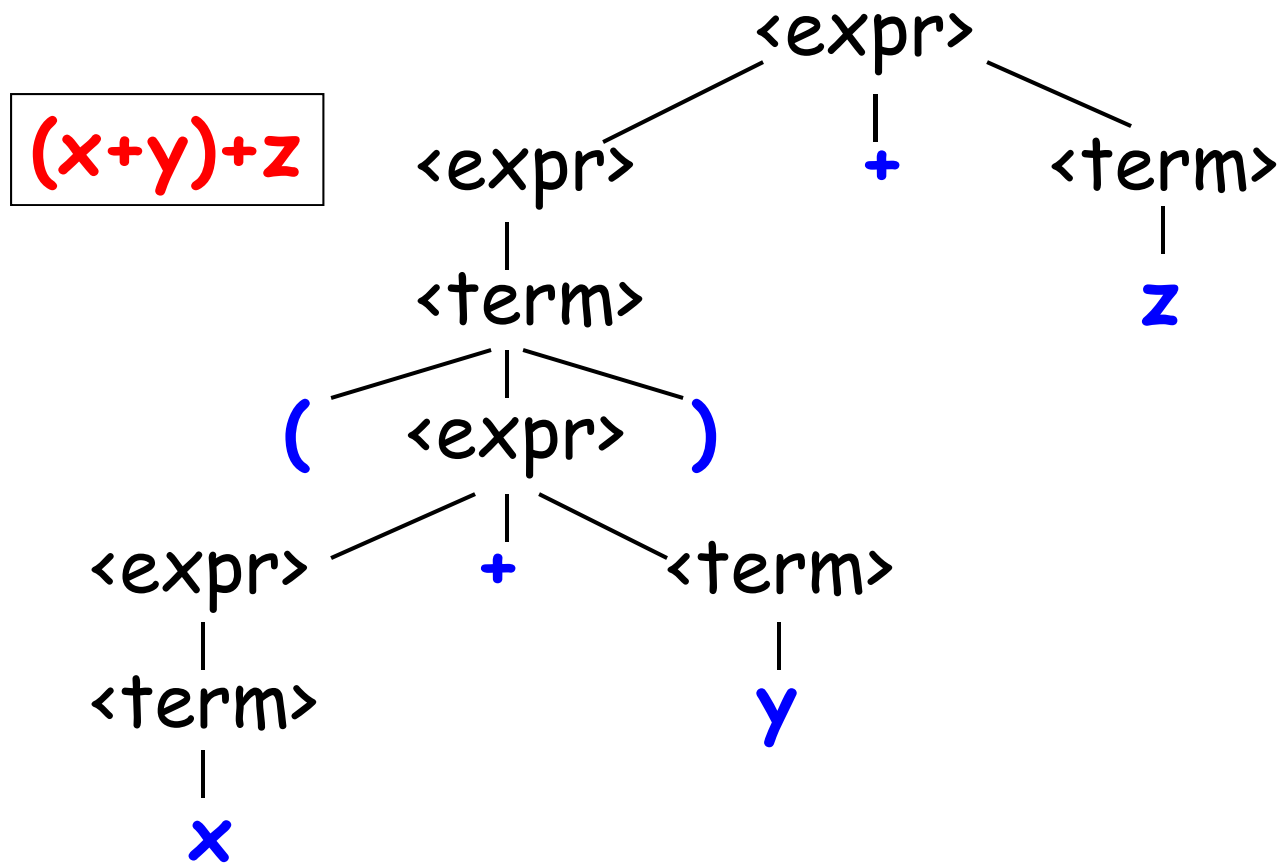
$\langle \text{exprs} \rangle ::= \langle \text{exp} \rangle \mid \langle \text{exprs} \rangle , \langle \text{exp} \rangle$

Derivation Tree

- Also called **parse tree** or **concrete syntax tree**
 - Leaf nodes: terminals
 - Inner nodes: non-terminals
 - Root: starting non-terminal of the grammar
- Describes a particular way to derive a string
 - Leaf nodes from left to right are the string
 - to get the string: depth-first traversal, following the leftmost unexplored branch

Example of a Derivation Tree

$\langle \text{expr} \rangle ::= \langle \text{term} \rangle \mid \langle \text{expr} \rangle + \langle \text{term} \rangle$
 $\langle \text{term} \rangle ::= x \mid y \mid z \mid (\langle \text{expr} \rangle)$



Equivalent Derivation Sequences

The set of string derivations that are represented by the same parse tree

One derivation:

$$\begin{aligned} \langle \text{expr} \rangle &\Rightarrow \langle \text{expr} \rangle + \langle \text{term} \rangle \Rightarrow \langle \text{expr} \rangle + z \Rightarrow \\ \langle \text{term} \rangle + z &\Rightarrow (\langle \text{expr} \rangle) + z \Rightarrow \\ (\langle \text{expr} \rangle + \langle \text{term} \rangle) + z &\Rightarrow (\langle \text{expr} \rangle + y) + z \Rightarrow \\ (\langle \text{term} \rangle + y) + z &\Rightarrow (x + y) + z \end{aligned}$$

Another derivation:

$$\begin{aligned} \langle \text{expr} \rangle &\Rightarrow \langle \text{expr} \rangle + \langle \text{term} \rangle \Rightarrow \langle \text{term} \rangle + \langle \text{term} \rangle \Rightarrow \\ (\langle \text{expr} \rangle) + \langle \text{term} \rangle &\Rightarrow (\langle \text{expr} \rangle + \langle \text{term} \rangle) + \langle \text{term} \rangle \Rightarrow \\ (\langle \text{term} \rangle + \langle \text{term} \rangle) + \langle \text{term} \rangle &\Rightarrow (x + \langle \text{term} \rangle) + \langle \text{term} \rangle \Rightarrow \\ (x + y) + \langle \text{term} \rangle &\Rightarrow (x + y) + z \end{aligned}$$

Many more ...

Ambiguous Grammars

- For some string, there are multiple different parse trees
- An ambiguous grammar gives more freedom to the compiler writer
 - e.g. for code optimizations
- For real-world programming languages, we typically have non-ambiguous grammars
 - To remove ambiguity: add non-terminals; or add operator precedence and associativity; or use an attribute grammar (more later ...)

Elimination of Ambiguity

$\langle \text{expr} \rangle ::= \langle \text{expr} \rangle + \langle \text{expr} \rangle \mid \langle \text{expr} \rangle * \langle \text{expr} \rangle \mid$
 $(\langle \text{expr} \rangle) \mid \text{id}$

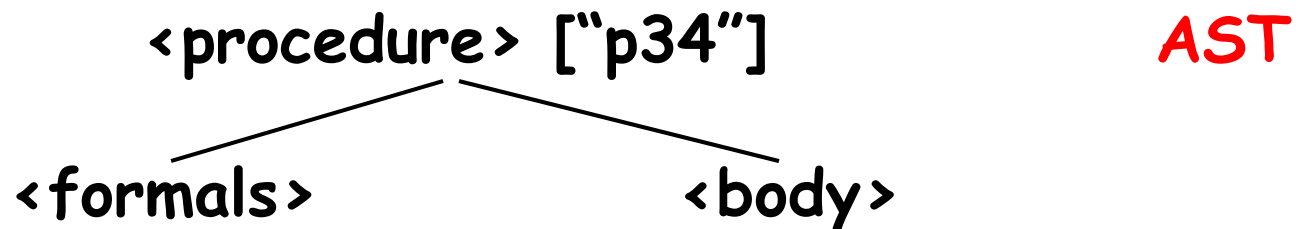
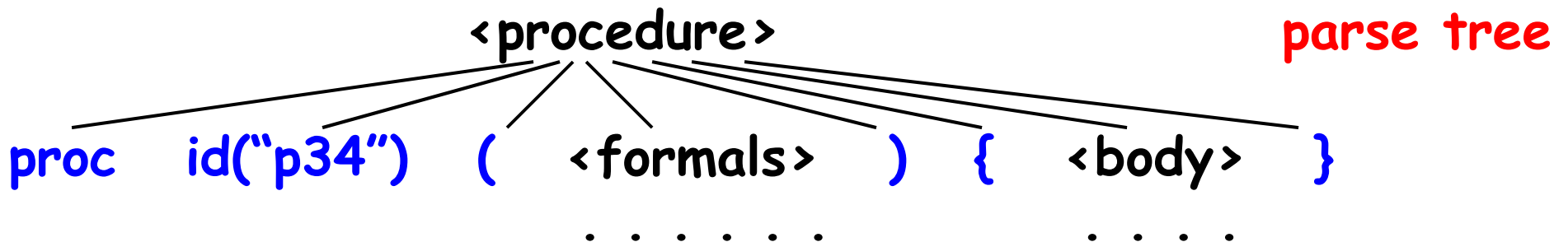
1. Prove that this grammar is ambiguous
2. Create an equivalent non-ambiguous grammar with the appropriate precedence and associativity
 - $*$ has higher precedence than $+$
 - both are left-associative

Example: what is the parse tree for
 $a + b * (c + d) * e$?

Abstract Syntax Trees (AST)

- A “simplified” version of a concrete syntax tree, without loss of information

$\langle \text{procedure} \rangle ::= \text{proc id (} \langle \text{formals} \rangle \text{) } \{ \langle \text{body} \rangle \}$



Use of Context-Free Grammars

- **Syntax** of a programming language
 - e.g. Java: Chapter 18 of the language specification (JLS) defines a grammar
 - Terminals: identifiers, keywords, literals, separators, operators
 - Starting non-terminal: **CompilationUnit**
- Implementation of a **parser** in a compiler
 - e.g. the JLS grammar (Ch. 18) is used by the parser inside the **javac** compiler

Limitations of Context-Free Grammars

- Cannot represent semantics
 - e.g. “every variable used in a statement should be declared earlier in the code”; or “the use of a variable should conform to its type” (type checking)
 - Need to allow only programs that satisfy certain **context-sensitive conditions**
- Cannot be used to generate things other than parse trees
 - e.g., what if we wanted to generate assembly code for the given program?

Attribute Grammars

- Generalization of context-free grammars
- Used for semantic checking and other compile-time analyses
 - e.g. type checking in a compiler
- Used for translation
 - e.g. parse tree → assembly code
- Represents a **traversal of the parse tree** and the computation of some information during that traversal

Structure of an Attribute Grammar

- Underlying context-free grammar
- For each terminal and non-terminal:
zero, one, or more **attributes**
 - For each attribute: domain of possible values
- Set of **evaluation rules** for each production
- Set of boolean **conditions** for attribute values

Example

- $L = \{ a^n b^n c^n \mid n > 0 \}$; not context-free

- BNF

$\langle \text{start} \rangle ::= \langle A \rangle \langle B \rangle \langle C \rangle$ $\langle A \rangle ::= a \mid a \langle A \rangle$

$\langle B \rangle ::= b \mid b \langle B \rangle$ $\langle C \rangle ::= c \mid c \langle C \rangle$

- Attributes

- **Na**: associated with $\langle A \rangle$
- **Nb**: associated with $\langle B \rangle$
- **Nc**: associated with $\langle C \rangle$
- Value domain for Na, Nb, Nc: integer values

Example: Alternative

- $L = \{ a^n b^n c^n \mid n > 0 \}$; not context-free

- BNF

$\langle \text{start} \rangle ::= \langle A \rangle \langle B \rangle \langle C \rangle$ $\langle A \rangle ::= a \mid a \langle A \rangle$
 $\langle B \rangle ::= b \mid b \langle B \rangle$ $\langle C \rangle ::= c \mid c \langle C \rangle$

- Attributes

- **N**: associated with $\langle A \rangle$, $\langle B \rangle$, $\langle C \rangle$
- Value domain for N: integer values

Example

- Evaluation rules (similar for $\langle B \rangle$, $\langle C \rangle$)

$\langle A \rangle ::= a$

$\langle A \rangle.Na := 1$

| $a\langle A \rangle_2$

$\langle A \rangle.Na := 1 + \langle A \rangle_2.Na$

- Conditions

$\langle \text{start} \rangle ::= \langle A \rangle \langle B \rangle \langle C \rangle$

Cond: $\langle A \rangle.Na = \langle B \rangle.Nb = \langle C \rangle.Nc$

Example: Alternative

- Evaluation rules (similar for $\langle B \rangle$, $\langle C \rangle$)

$\langle A \rangle ::= a$

$\langle A \rangle.N := 1$

| $a\langle A \rangle_2$

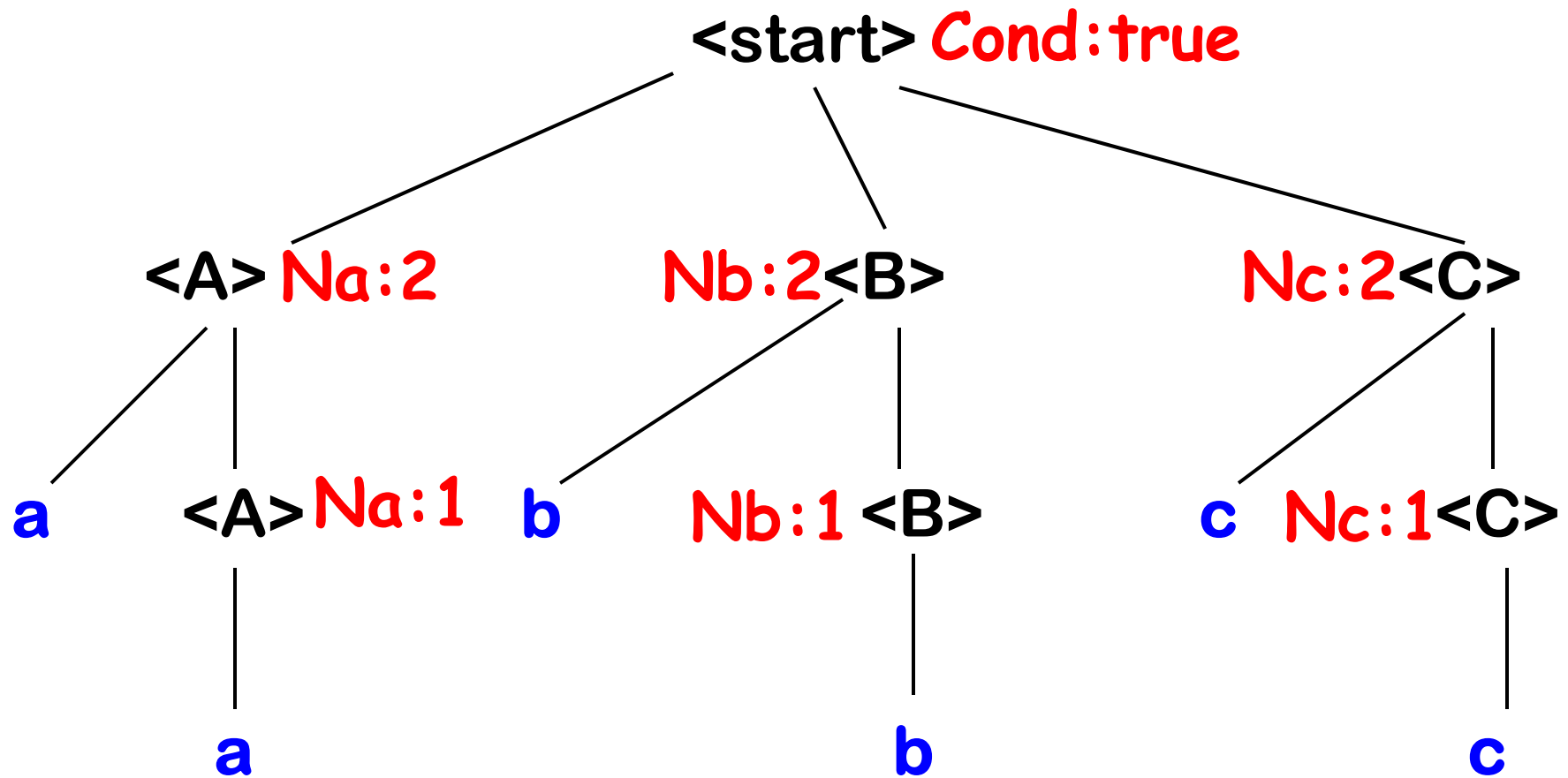
$\langle A \rangle.N := 1 + \langle A \rangle_2.N$

- Conditions

$\langle \text{start} \rangle ::= \langle A \rangle \langle B \rangle \langle C \rangle$

Cond: $\langle A \rangle.N = \langle B \rangle.N = \langle C \rangle.N$

Parse Tree



Parse Tree for an Attribute Grammar

- Valid tree for the underlying BNF
- Each node has (attribute,value) pairs
 - One pair for each attribute associated with the terminal or non-terminal in the node
- Some nodes have boolean conditions
- **Valid** parse tree
 - Attribute values conform to the evaluation rules
 - All boolean conditions are true

Synthesized vs. Inherited Attributes

- Each terminal and non-terminal X : disjoint sets of attributes $\text{Syn}(X)$ and $\text{Inh}(X)$
 - $\text{Attr}(X) = \text{Syn}(X) \cup \text{Inh}(X)$
- Production $X ::= Y_1 Y_2 \dots Y_n$
- Each attribute in $\text{Syn}(X)$ is defined from $\text{Attr}(Y_1) \cup \dots \cup \text{Attr}(Y_n) \cup \text{Attr}(X)$
- Each attribute in $\text{Inh}(Y_k)$ is defined from $\text{Attr}(X) \cup \text{Attr}(Y_1) \cup \dots \cup \text{Attr}(Y_n)$

Complete Evaluation Rules

- Synthesized attribute associated with N:
 - Each alternative in “ $N ::= \dots$ ” should contain a rule for evaluating the attribute
- Inherited attribute associated with N:
 - For every occurrence of N in “ $\dots ::= \dots N \dots$ ” there must be a rule for evaluating the attribute
- Whenever you create an attribute grammar (in homeworks/exams), make sure it satisfies these requirements

Example: Binary Numbers

- Context-free grammar

$\langle B \rangle ::= \langle D \rangle$

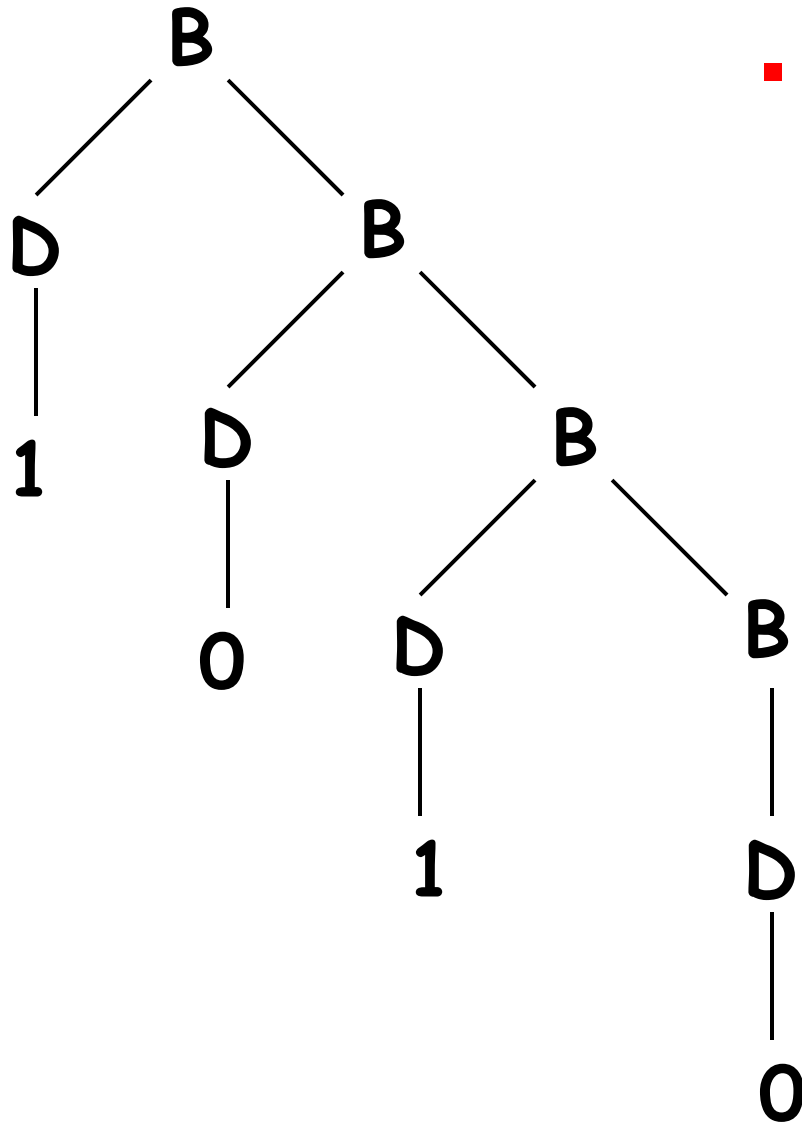
$\langle B \rangle ::= \langle D \rangle \langle B \rangle$

$\langle D \rangle ::= 0$

$\langle D \rangle ::= 1$

- Goal: compute the **value** of a binary number

BNF Parse Tree for Input 1010



- Add attributes
 - ****: synthesized **val**
 - ****: synthesized **pos**
 - **<D>**: inherited **pow**
 - **<D>**: **val**

Example: Binary Numbers

$\langle B \rangle ::= \langle D \rangle$

$\langle B \rangle.\text{pos} := 1$

$\langle B \rangle.\text{val} := \langle D \rangle.\text{val}$

$\langle D \rangle.\text{pow} := 0$

$\langle B \rangle_1 ::= \langle D \rangle \langle B \rangle_2$

$\langle B \rangle_1.\text{pos} := \langle B \rangle_2.\text{pos} + 1$

$\langle B \rangle_1.\text{val} := \langle B \rangle_2.\text{val} + \langle D \rangle.\text{val}$

$\langle D \rangle.\text{pow} := \langle B \rangle_2.\text{pos}$

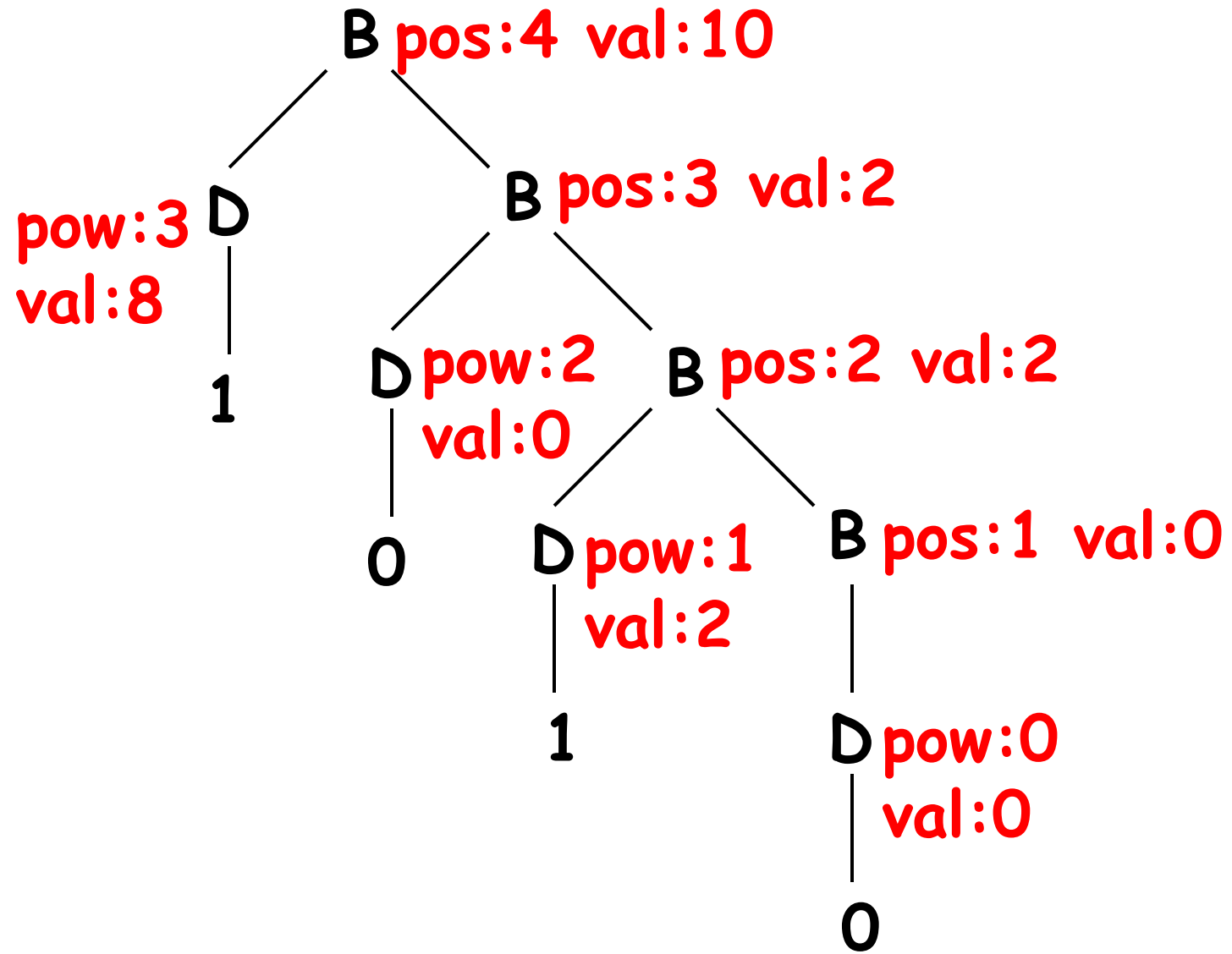
$\langle D \rangle ::= 0$

$\langle D \rangle.\text{val} := 0$

$\langle D \rangle ::= 1$

$\langle D \rangle.\text{val} := 2^{\langle D \rangle.\text{pow}}$

Evaluated Parse Tree



Example: Expression Language

- Problem: evaluate an expression

$\langle S \rangle ::= \langle E \rangle$

$\langle E \rangle ::= 0 \mid 1 \mid \langle I \rangle \mid (\langle E \rangle + \langle E \rangle) \mid$
 $\text{let } \langle I \rangle = \langle E \rangle \text{ in } \langle E \rangle \text{ end}$

$\langle I \rangle ::= \text{id}$

- Attributes
 - $\langle I \rangle$: synthesized **name**
 - $\langle E \rangle$: synthesized **val**, inherited **env**
 - **id**: synthesized **lexval**, represents the string that the lexical analysis puts inside token **id**

Attribute Grammar

$\langle S \rangle ::= \langle E \rangle$

$\langle E \rangle.\text{env} := \emptyset$

$\langle E \rangle ::= 0$

$\langle E \rangle.\text{val} := 0$

$\langle E \rangle ::= 1$

$\langle E \rangle.\text{val} := 1$

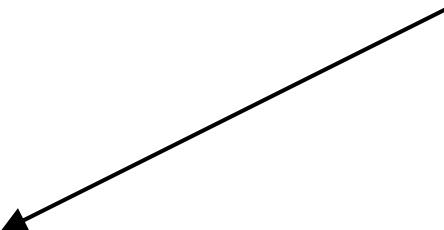
$\langle E \rangle ::= \langle l \rangle$

$\langle E \rangle.\text{val} := \text{lookup}(\langle E \rangle.\text{env}, \langle l \rangle.\text{name})$

$\langle l \rangle ::= \text{id}$

$\langle l \rangle.\text{name} := \text{id.lexval}$

"helper" function



Attribute Grammar

$\langle E \rangle_1 ::= (\langle E \rangle_2 + \langle E \rangle_3)$

$\langle E \rangle_1.\text{val} := \langle E \rangle_2.\text{val} + \langle E \rangle_3.\text{val}$

$\langle E \rangle_2.\text{env} := \langle E \rangle_1.\text{env}$

$\langle E \rangle_3.\text{env} := \langle E \rangle_1.\text{env}$

$\langle E \rangle_1 ::= \text{let } \langle I \rangle = \langle E \rangle_2 \text{ in } \langle E \rangle_3 \text{ end}$

$\langle E \rangle_2.\text{env} := \langle E \rangle_1.\text{env}$

$\langle E \rangle_3.\text{env} :=$

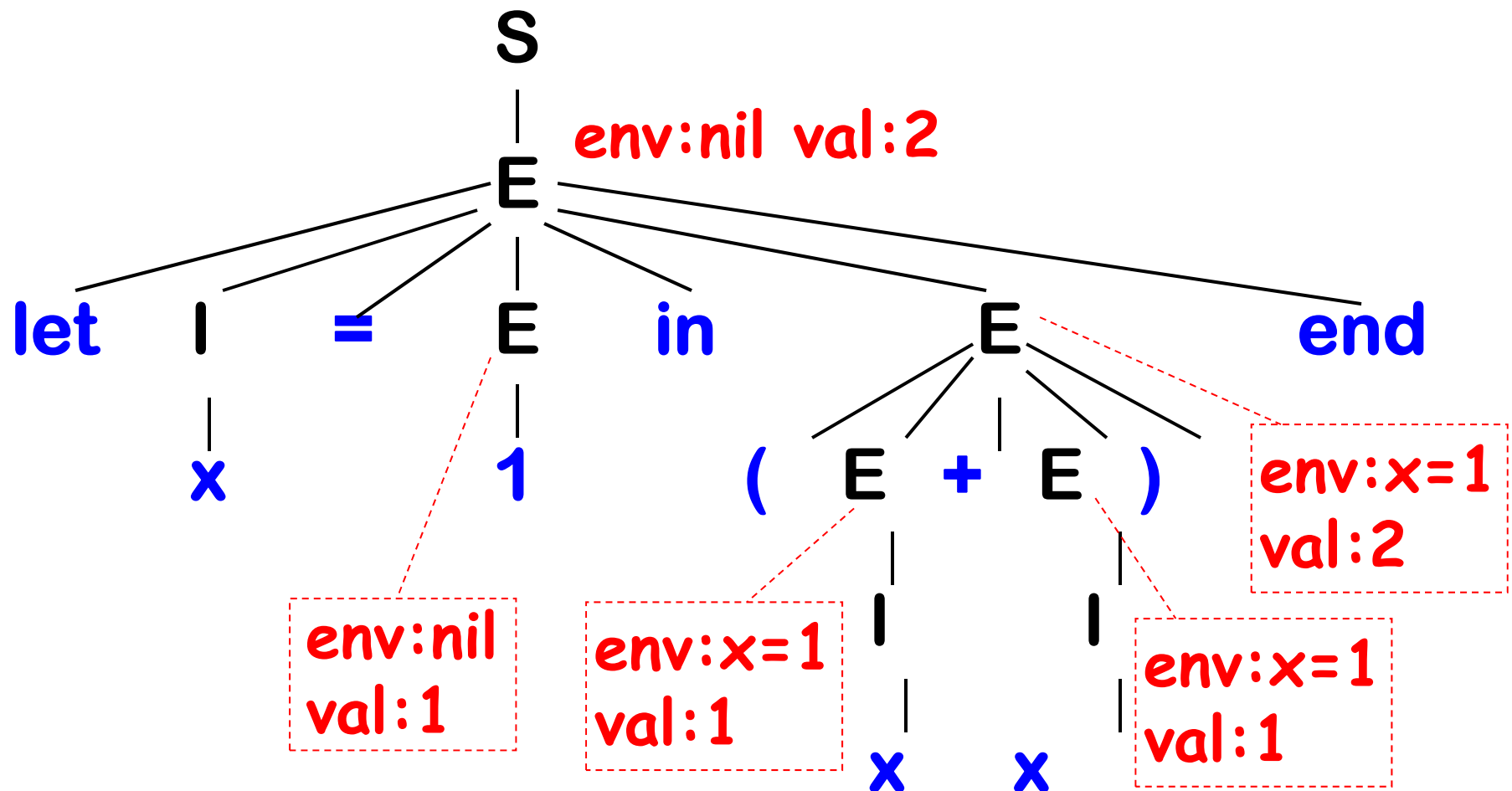
$\text{update}(\langle E \rangle_1.\text{env}, \langle I \rangle.\text{name}, \langle E \rangle_2.\text{val})$

$\langle E \rangle_1.\text{val} := \langle E \rangle_3.\text{val}$

creates a
"fresh"
environment;
does not
affect
 $\langle E \rangle_1.\text{env}$

Example

- Evaluation of **let x = 1 in (x+x) end**



Another Example

- How about the evaluation of

let $x = 1$ in

let $x = (x+1)$ in

$(x+2)$

end

end

More than Context-Free Power

- $L = \{ a^n b^n c^n \mid n > 0 \}$
 - Unlike $L = \{ a^n b^n \mid n > 0 \}$, here we need explicit counting
- $L = \{ w c w \mid w \in \{a,b\}^* \}$
 - The “flavor” of checking whether identifiers are declared before their uses
 - Cannot be done with a context-free grammar – requires context-sensitive conditions
- Syntax analysis (i.e., parser) cannot handle semantic properties

“Fixing” Context-Free Grammars

- $\langle S \rangle_1 ::= a \mid b \mid \langle S \rangle_2 \langle S \rangle_3$
 - Ambiguous: e.g. abab can be parsed as if it were (ab)(ab) or a(b(a(b))) or ...
 - Suppose we want to make the grammar unambiguous, and associate to the right
- One approach: $\langle X \rangle ::= a \mid b$
 $\langle S \rangle ::= \langle X \rangle \langle S \rangle \mid \langle X \rangle$
- Another approach: attribute **len** for S
 - $\langle S \rangle ::= a \mid b$ $\langle S \rangle.\text{len} := 1$
 - $\langle S \rangle_1 ::= \langle S \rangle_2 \langle S \rangle_3$ $\langle S \rangle_1.\text{len} := \langle S \rangle_2.\text{len} + \langle S \rangle_3.\text{len}$
Cond: $\langle S \rangle_2.\text{len} = 1$

“Fixing” Context-Free Grammars

- $\langle E \rangle_1 ::= \langle \text{Num} \rangle \mid \langle E \rangle_2 + \langle E \rangle_3 \leftarrow \text{ambiguous}$
 - Want left-associativity:
 - $a+b+c$ as if it were $(a+b)+c$, not $a+(b+c)$
 - Change BNF: $\langle E \rangle_1 ::= \langle \text{Num} \rangle \mid \langle E \rangle_2 + \langle \text{Num} \rangle$
 - Alternative: attribute grammar
 - Attribute n : number of $+$
 - $\langle E \rangle_1 ::= \langle \text{Num} \rangle \quad \langle E \rangle_1.n := 0$
 - $\langle E \rangle_1 ::= \langle E \rangle_2 + \langle E \rangle_3 \quad \langle E \rangle_1.n := 1 + \langle E \rangle_2.n + \langle E \rangle_3.n$
- Cond: ?

Attribute Evaluation: Dependence Graph

- $\langle A \rangle.x := \langle B \rangle.y + \langle C \rangle.z$
 - $\langle A \rangle.x$ depends on $\langle B \rangle.y$
 - $A.x \leftarrow B.y$ dependence edge
 - $\langle A \rangle.x$ depends on $\langle C \rangle.z$
 - $A.x \leftarrow C.z$ dependence edge
- $\langle A \rangle_1.x := \langle A \rangle_2.x$
 - $\langle A \rangle_1.x$ depends on $\langle A \rangle_2.x$
 - $A_1.x \leftarrow A_2.x$ dependence edge

Attribute Grammar Evaluation

- Given: parse tree for parsed input with attributes attached to tree nodes
- Find evaluation order of attributes
 - Build **dependence graph**
 - Node: a pair (parse tree node, attribute)
 - Complain about cycles in the graph
 - Topologically sort the graph
- Evaluate the attributes in sorted order

Example: Binary Numbers

$\langle B \rangle ::= \langle D \rangle$

$\langle B \rangle.\text{pos} := 1$

$\langle B \rangle.\text{val} := \langle D \rangle.\text{val}$

$\langle D \rangle.\text{pow} := 0$

$\langle B \rangle_1 ::= \langle D \rangle \langle B \rangle_2$

$\langle B \rangle_1.\text{pos} := \langle B \rangle_2.\text{pos} + 1$

$\langle B \rangle_1.\text{val} := \langle B \rangle_2.\text{val} + \langle D \rangle.\text{val}$

$\langle D \rangle.\text{pow} := \langle B \rangle_2.\text{pos}$

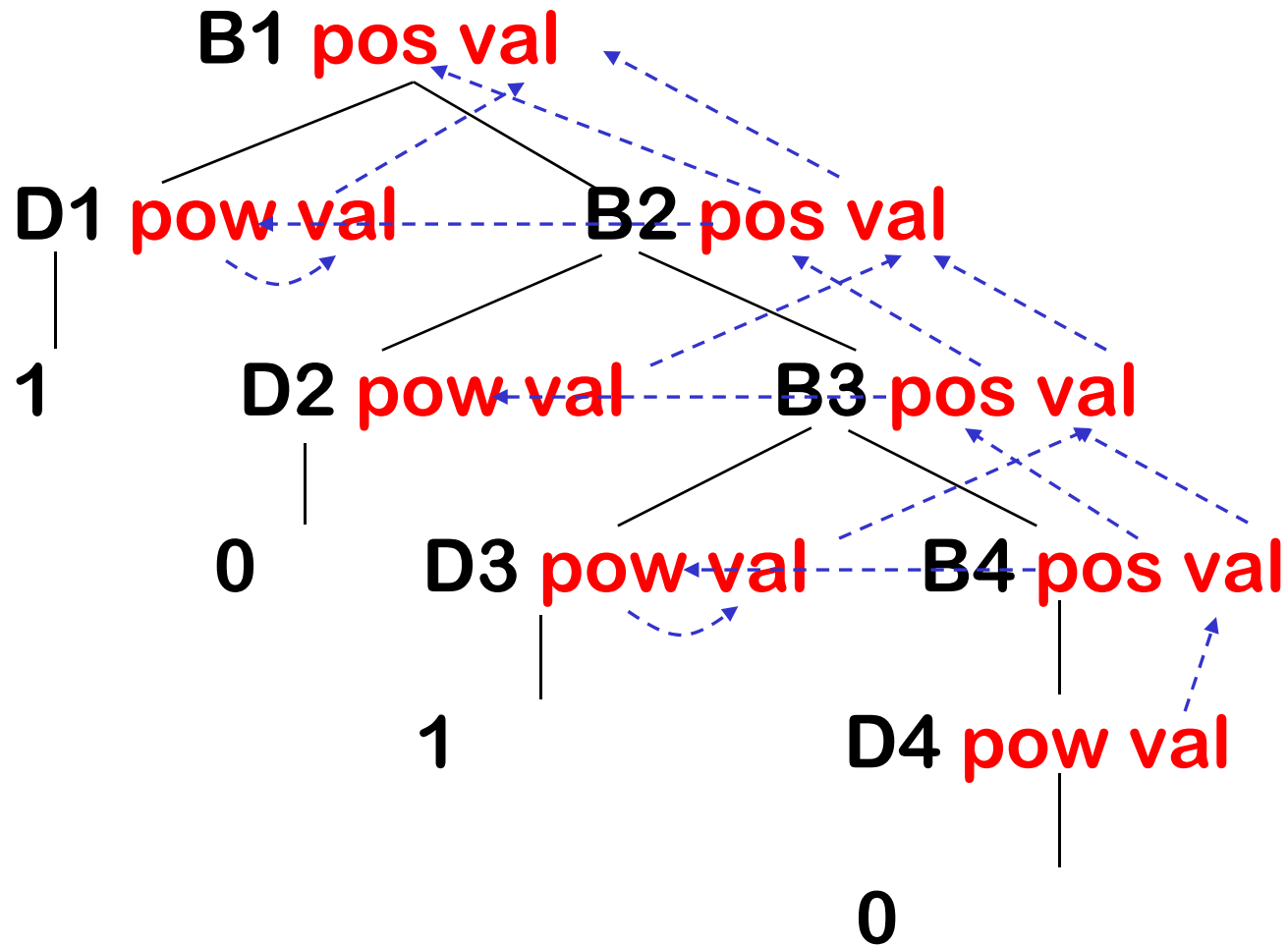
$\langle D \rangle ::= 0$

$\langle D \rangle.\text{val} := 0$

$\langle D \rangle ::= 1$

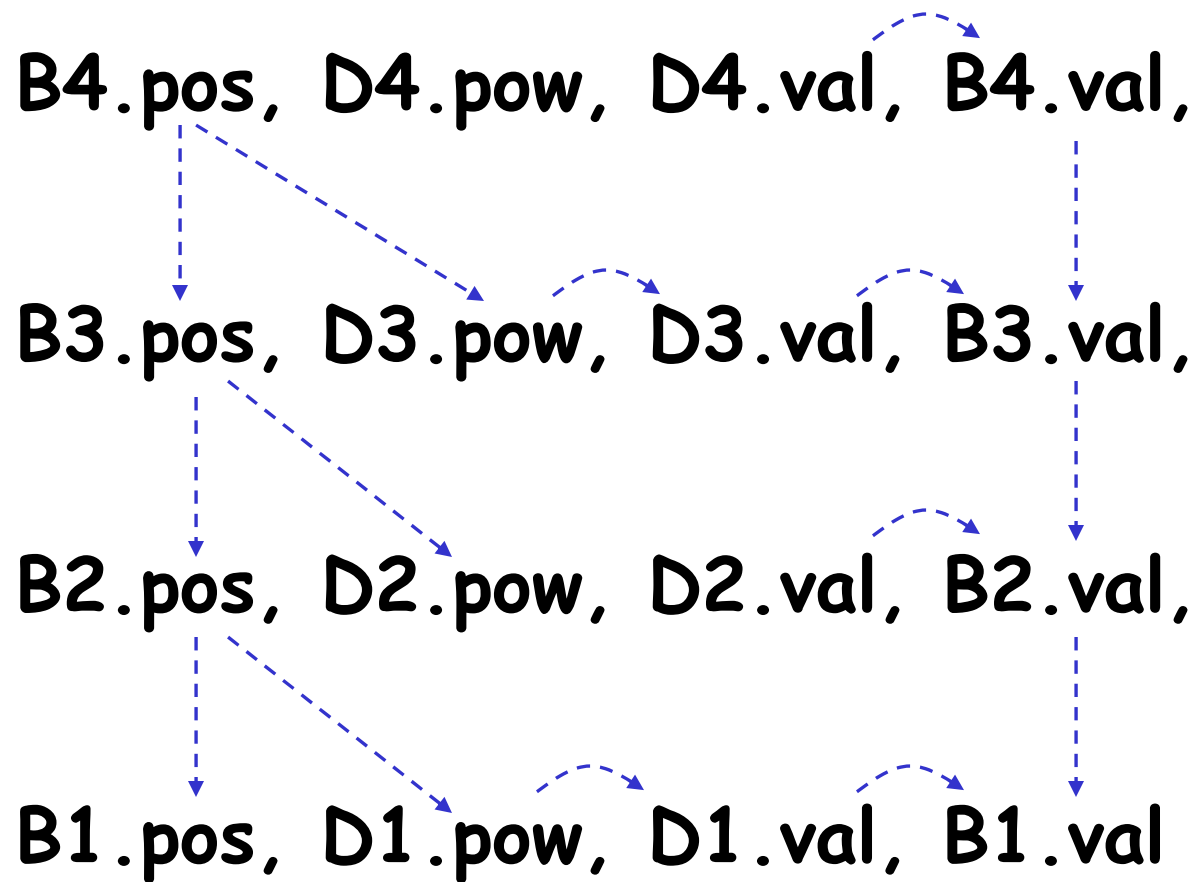
$\langle D \rangle.\text{val} := 2^{\langle D \rangle.\text{pow}}$

Dependence Graph for Binary Numbers



Sort the Graph

- Topological sort: x is “smaller” than y iff $x \rightarrow y$



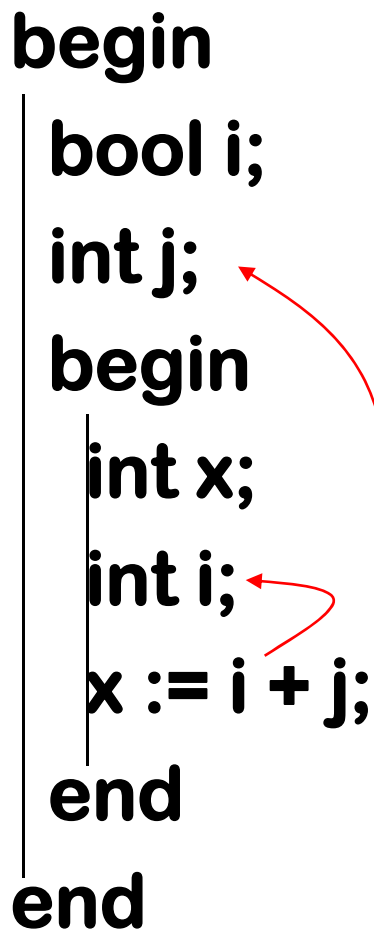
Cycles

- The notion of “topological sort” only makes sense for directed acyclic graphs
- Cycles in the dependence graph: recursive dependencies
 - There are approaches to solve meaningful recursive systems of equations
 - e.g., fixed-point computation
- In this course we will disallow cycles
 - No cyclic dependencies in your solutions in exams and homeworks

Long Example 1: Type Checking

- Given: program with declarations
- Check that variables are used correctly

```
begin
  bool i;
  int j;
  begin
    int x;
    int i;
    x := i + j;
  end
end
```



Blocks, Declarations, Statements

$\langle \text{prog} \rangle ::= \langle \text{block} \rangle$

$\langle \text{block} \rangle ::= \text{begin } \langle \text{declist} \rangle ; \langle \text{stmtlist} \rangle \text{ end}$

$\langle \text{stmtlist} \rangle ::= \langle \text{stmt} \rangle$

$| \langle \text{stmt} \rangle ; \langle \text{stmtlist} \rangle$

$\langle \text{stmt} \rangle ::= \langle \text{assign} \rangle$

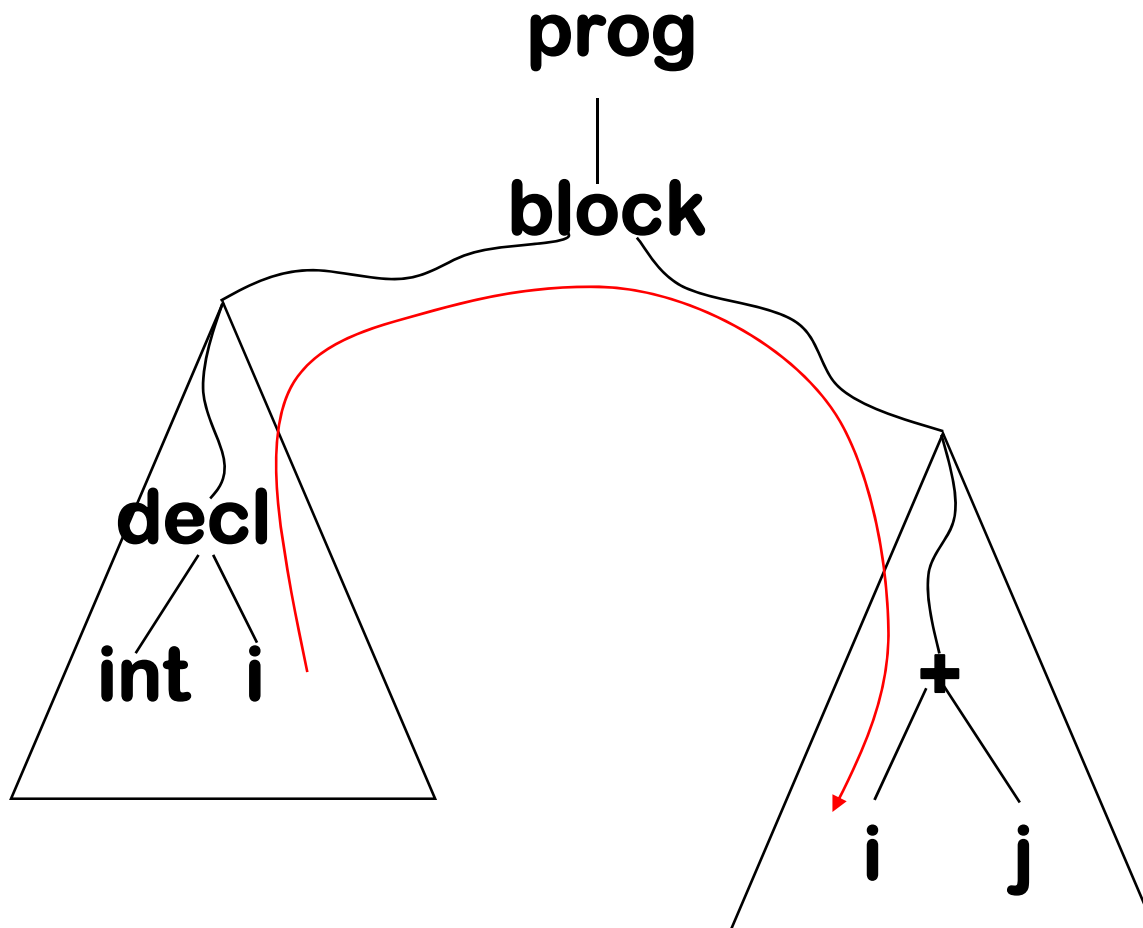
$| \langle \text{block} \rangle$

$| \dots$

Problem

- Check types of variables: int and bool
- For nested blocks, use the innermost declaration
- Check parameter types and return type for functions
 - For simplicity: all functions will return int
- Type checking: a form of semantic checking that a compiler will perform on the parse tree (or the AST)

Parse Tree



Data Structure

- Use a **stack** of **symbol tables**
 - symbol table: set of pairs (name,type)
- Build a symbol table for the declarations in a begin-end block
 - as a synthesized attribute **tbl**
- “Propagate” a stack of symbol tables to statements and expressions
 - propagate downwards on the parse tree as an inherited attribute **alltbl**

Type Checking

- Pass around a stack of symbol tables

begin

bool i;

int j;

begin

int x;

int i;

x := i + j;

end

end

[
{"x",INT), ("i",INT)},
{"i",BOOL), ("j",INT)}
]

bottom of stack

Context-Free Grammar – Part 1

$\langle \text{prog} \rangle ::= \langle \text{block} \rangle$

$\langle \text{block} \rangle ::= \text{begin } \langle \text{declist} \rangle ; \langle \text{stmtlist} \rangle \text{ end}$

$\langle \text{declist} \rangle ::= \langle \text{decl} \rangle \mid \langle \text{decl} \rangle ; \langle \text{declist} \rangle$

$\langle \text{decl} \rangle ::= \text{int id} \mid \text{bool id} \mid$

$\text{fun id } (\langle \text{formalslist} \rangle) : \text{int} = \langle \text{block} \rangle$

$\langle \text{stmtlist} \rangle ::= \langle \text{stmt} \rangle \mid \langle \text{stmt} \rangle ; \langle \text{stmtlist} \rangle$

$\langle \text{stmt} \rangle ::= \langle \text{assign} \rangle \mid \langle \text{block} \rangle \mid$

$\text{if } \langle \text{boolexpr} \rangle$

$\text{then } \langle \text{stmtlist} \rangle \text{ else } \langle \text{stmtlist} \rangle \text{ fi}$

Context-Free Grammar – Part 2

$\langle \text{assign} \rangle ::= \text{id} := \langle \text{boolexp} \rangle \mid$
 $\text{id} := \langle \text{intexp} \rangle$

$\langle \text{boolexp} \rangle ::= \text{true} \mid \text{false} \mid \text{id}$

$\langle \text{intexp} \rangle ::= \text{intconst} \mid \text{id} \mid$
 $\langle \text{intexp} \rangle + \langle \text{intexp} \rangle \mid$
 $\text{id} (\langle \text{actualslist} \rangle)$

Ignore Functions for Now

$\langle \text{prog} \rangle ::= \langle \text{block} \rangle$

$\langle \text{block} \rangle.\text{alltbl} := \text{emptystack}$

$\langle \text{block} \rangle ::= \text{begin } \langle \text{declist} \rangle ; \langle \text{stmtlist} \rangle \text{ end}$

*creates
a "fresh"
stack*

$\langle \text{stmtlist} \rangle.\text{alltbl} :=$

$\text{push}(\langle \text{declist} \rangle.\text{tbl}, \langle \text{block} \rangle.\text{alltbl})$

$\langle \text{stmtlist} \rangle_1 ::= \langle \text{stmt} \rangle$

$\langle \text{stmt} \rangle.\text{alltbl} := \langle \text{stmtlist} \rangle_1.\text{alltbl}$

$| \langle \text{stmt} \rangle ; \langle \text{stmtlist} \rangle_2$

$\langle \text{stmt} \rangle.\text{alltbl} := \langle \text{stmtlist} \rangle_1.\text{alltbl}$

$\langle \text{stmtlist} \rangle_2.\text{alltbl} := \langle \text{stmtlist} \rangle_1.\text{alltbl}$

Declarations

$\langle \text{declist} \rangle_1 ::= \langle \text{decl} \rangle$

$\langle \text{declist} \rangle_1.\text{tbl} := \langle \text{decl} \rangle.\text{tbl}$

$| \langle \text{decl} \rangle ; \langle \text{declist} \rangle_2$

$\langle \text{declist} \rangle_1.\text{tbl} :=$

$\langle \text{decl} \rangle.\text{tbl} \cup \langle \text{declist} \rangle_2.\text{tbl}$

Cond:

$\text{ids}(\langle \text{decl} \rangle.\text{tbl}) \cap \text{ids}(\langle \text{declist} \rangle_2.\text{tbl}) = \emptyset$

ids is a function that returns the set of all names in a table;

used to check for duplicates: e.g. `int x; int x;`

Declarations

<decl> ::= int id

<decl>.tbl := { (id.lexval, INT) }

| bool id

<decl>.tbl := { (id.lexval, BOOL) }

| fun id (<formalslist>) : int = <block>

ignore for now



Statements

$\langle \text{stmtlist} \rangle_1 ::= \langle \text{stmt} \rangle$

$\langle \text{stmt} \rangle.\text{alltbl} := \langle \text{stmtlist} \rangle_1.\text{alltbl}$

| $\langle \text{stmt} \rangle ; \langle \text{stmtlist} \rangle_2$

$\langle \text{stmt} \rangle.\text{alltbl} := \langle \text{stmtlist} \rangle_1.\text{alltbl}$

$\langle \text{stmtlist} \rangle_2.\text{alltbl} := \langle \text{stmtlist} \rangle_1.\text{alltbl}$

$\langle \text{stmt} \rangle ::= \langle \text{assign} \rangle$

$\langle \text{assign} \rangle.\text{alltbl} := \langle \text{stmt} \rangle.\text{alltbl}$

| $\langle \text{block} \rangle$

$\langle \text{block} \rangle.\text{alltbl} := \langle \text{stmt} \rangle.\text{alltbl}$

| **if** $\langle \text{boolexp} \rangle$ **then** $\langle \text{stmtlist} \rangle$ **else** ...

$\langle \text{boolexp} \rangle.\text{alltbl} := \langle \text{stmt} \rangle.\text{alltbl}$

$\langle \text{stmtlist} \rangle.\text{alltbl} := \langle \text{stmt} \rangle.\text{alltbl}$

Statements

<assign> ::= id := <intexp>

<intexp>.alltbl := <assign>.alltbl

the "last"
type on
the stack

Cond:

typeof(id.lexval, <assign>.alltbl) = INT

| id := <boolexp>

<boolexp>.alltbl := <assign>.alltbl

Cond:

typeof(id.lexval, <assign>.alltbl) = BOOL

Expressions

<intexp>₁ ::= intconst
| **id**
Cond: typeof(**id**.lexval,
 <intexp>₁.alltbl) = INT
| **<intexp>₂ + <intexp>₃**
 <intexp>₂.alltbl := <intexp>₁.alltbl
 <intexp>₃.alltbl := <intexp>₁.alltbl

<boolexp> ::= true
| **false**
| **id**
Cond: typeof(**id**.lexval,
 <boolexp>.alltbl) = BOOL

Declarations

$\langle \text{decl} \rangle ::= \text{int id}$

$\langle \text{decl} \rangle.\text{tbl} := \{ (\text{id}.\text{lexval}, \text{INT}) \}$

| bool id

$\langle \text{decl} \rangle.\text{tbl} := \{ (\text{id}.\text{lexval}, \text{BOOL}) \}$

| $\text{fun id} (\langle \text{formalslist} \rangle) : \text{int} = \langle \text{block} \rangle$

$\langle \text{decl} \rangle.\text{tbl} :=$

$\{ (\text{id}.\text{lexval},$

$\text{FUN}(\langle \text{formalslist} \rangle.\text{types}, \text{INT})) \}$

synthesized attr:
ordered list of INT/BOOL

$\langle \text{block} \rangle.\text{alltbl} \rightarrow$ what are we going
to do here?

More Propagation of Information

- **Declarations:** add inherited **alltbl**

$\langle \text{block} \rangle ::= \text{begin } \langle \text{declist} \rangle ; \langle \text{stmtlist} \rangle \text{ end}$

$\langle \text{stmtlist} \rangle.\text{alltbl} :=$

$\text{push}(\langle \text{declist} \rangle.\text{tbl}, \langle \text{block} \rangle.\text{alltbl})$

$\langle \text{declist} \rangle.\text{alltbl} :=$

$\text{push}(\langle \text{declist} \rangle.\text{tbl}, \langle \text{block} \rangle.\text{alltbl})$

We first gather the declarations in $\langle \text{declist} \rangle$, and then we use them to check the blocks "embedded" in $\langle \text{declist} \rangle$

Declarations

$\langle \text{declist} \rangle_1 ::= \langle \text{decl} \rangle$

$\langle \text{declist} \rangle_1.\text{tbl} := \langle \text{decl} \rangle.\text{tbl}$

$\langle \text{decl} \rangle.\text{alltbl} := \langle \text{declist} \rangle_1.\text{alltbl}$

$| \langle \text{decl} \rangle ; \langle \text{declist} \rangle_2$

$\langle \text{declist} \rangle_1.\text{tbl} :=$

$\langle \text{decl} \rangle.\text{tbl} \cup \langle \text{declist} \rangle_2.\text{tbl}$

$\langle \text{decl} \rangle.\text{alltbl} := \langle \text{declist} \rangle_1.\text{alltbl}$

$\langle \text{declist} \rangle_2.\text{alltbl} := \langle \text{declist} \rangle_1.\text{alltbl}$

Cond:

$\text{ids}(\langle \text{decl} \rangle.\text{tbl}) \cap \text{ids}(\langle \text{declist} \rangle_2.\text{tbl}) = \emptyset$

Declarations Again

```
<decl> ::= int id
        <decl>.tbl := { (id.lexval, INT) }
| bool id
  <decl>.tbl := { (id.lexval, BOOL) }
| fun id ( <formalslist> ) : int = <block>
  <decl>.tbl :=
    { (id.lexval,
      FUN(<formalslist>.types, INT) ) }
  <block>.alltbl :=
    push(<formalslist>.tbl, <decl>.alltbl)
```

Type-Checking Function Bodies

- Is the following code legal?

```
begin
  fun f (int i): int =
    begin
      ... g(5) ...
    end;
  fun g (int j): int =
    begin
      ...
    end; ...
end
```

Function Call

<actualslist> is
a list of <intexp>
and <boolexp>

<intexp> ::= **id** (<actualslist>)

redundant
if we have
only int return
types

Cond: typeof(**id**.lexval,
<intexp>.**alltbl**) = FUN

Cond: rettype(**id**.lexval,
<intexp>.**alltbl**) = INT

<actualslist>.**expTypes** :=
paramtypes(**id**.lexval,
<intexp>.**alltbl**)

<actualslist>.**alltbl** :=
<intexp>.**alltbl**

Long Example 2: Code Generation

- **Given: parse tree for a simple program (after type checking)**
- **Translate to assembly code**
- **The evaluation rules of the attribute grammar generate the assembly code**

Simple Imperative Language (IMP)

$\langle c \rangle_1 ::= \text{skip} \mid \langle \text{assign} \rangle \mid \langle c \rangle_2 ; \langle c \rangle_3$
 $\mid \text{if } \langle \text{be} \rangle \text{ then } \langle c \rangle_2 \text{ else } \langle c \rangle_3$
 $\mid \text{while } \langle \text{be} \rangle \text{ do } \langle c \rangle_2$

$\langle \text{assign} \rangle ::= \text{id} := \langle \text{ae} \rangle$

$\langle \text{ae} \rangle_1 ::= \text{id} \mid \text{intconst} \mid \langle \text{ae} \rangle_2 + \langle \text{ae} \rangle_3$
 $\mid \langle \text{ae} \rangle_2 - \langle \text{ae} \rangle_3 \mid \langle \text{ae} \rangle_2 * \langle \text{ae} \rangle_3$

$\langle \text{be} \rangle_1 ::= \text{true} \mid \text{false}$
 $\mid \langle \text{ae} \rangle_1 = \langle \text{ae} \rangle_2 \mid \langle \text{ae} \rangle_1 < \langle \text{ae} \rangle_2$
 $\mid \neg \langle \text{be} \rangle_2 \mid \langle \text{be} \rangle_2 \wedge \langle \text{be} \rangle_3$
 $\mid \langle \text{be} \rangle_2 \vee \langle \text{be} \rangle_3$

Assembly Language

- Processor with a single register (“accumulator”)
- **LOAD x**: copy the value of memory location (variable) x into the accumulator
- **LOAD const**: set the value of the accumulator to an integer constant
- **STO x**: write accumulator to memory location (variable) x

Assembly Language

- **ADD x**: add the value of memory location x to the content of the accumulator
 - The result stays in the accumulator
- **ADD const**: add constant to accumulator
- **BR L**: branch to label L (goto)
- **BZ L**: if accumulator is zero, branch to label L
- **L: NOP**: label L, associated with a “no-operation” instruction NOP

Code Generation Strategy

- Synthesized attribute **code**
 - Contains a sequence of instructions

- Example:

$\langle ae \rangle_1 ::= \langle ae \rangle_2 + \langle ae \rangle_3$
 $\langle ae \rangle_1.\text{code} :=$

leaves its result
in the accumulator

$\langle$
 $\langle ae \rangle_2.\text{code},$
 "STO T1",
 $\langle ae \rangle_3.\text{code},$
 "ADD T1"
 $>$

temp
var

Code Generation Strategy

- $\langle c \rangle_1 ::= \text{if } \langle be \rangle \text{ then } \langle c \rangle_2 \text{ else } \langle c \rangle_3$
 $\langle c \rangle_1.\text{code} := <$
 $\langle be \rangle.\text{code},$
 "BZ L1",
 $\langle c \rangle_2.\text{code},$
 "BR L2",
 "L1: NOP",
 $\langle c \rangle_3.\text{code},$
 "L2: NOP"
 $>$

Problems

- T1 cannot be used in $\langle ae \rangle_3.code$
 - Need to generate temporary names
- Labels L1 and L2 cannot be used in $\langle c \rangle_2.code$, $\langle c \rangle_3.code$, or elsewhere
 - Need to generate label names
- Keep counter for temporary names
 - inherited **temp**
- Keep “global” counter for label names
 - inherited **labin**, synthesized **labout**

Code Generation for Statements

$\langle \text{prog} \rangle ::= \langle c \rangle$

$\langle \text{prog} \rangle.\text{code} := \langle c \rangle.\text{code}$

$\langle c \rangle.\text{labin} := 0$

$\langle c \rangle_1 ::= \langle c \rangle_2 ; \langle c \rangle_3$

$\langle c \rangle_1.\text{code} :=$

$\text{concat}(\langle c \rangle_2.\text{code}, \langle c \rangle_3.\text{code})$

$\langle c \rangle_2.\text{labin} := \langle c \rangle_1.\text{labin}$

$\langle c \rangle_3.\text{labin} := \langle c \rangle_2.\text{labout}$

$\langle c \rangle_1.\text{labout} := \langle c \rangle_3.\text{labout}$

Code Generation for Statements

$\langle c \rangle ::= \text{skip}$

$\langle c \rangle.\text{code} := \langle \text{"NOP"} \rangle$

$\langle c \rangle.\text{labout} := \langle c \rangle.\text{labin}$

| $\langle \text{assign} \rangle$

$\langle c \rangle.\text{code} := \langle \text{assign} \rangle.\text{code}$

$\langle c \rangle.\text{labout} := \langle c \rangle.\text{labin}$

Code Generation for Statements

$\langle c \rangle_1 ::= \text{if } \langle be \rangle \text{ then } \langle c \rangle_2 \text{ else } \langle c \rangle_3$

$\langle c \rangle_2.labin := \langle c \rangle_1.labin + 2$

$\langle c \rangle_3.labin := \langle c \rangle_2.labout$

$\langle c \rangle_1.labout := \langle c \rangle_3.labout$

$\langle c \rangle_1.code := \text{concat}(\langle be \rangle.code,$
 ("BZ" label($\langle c \rangle_1.labin$)),
 $\langle c \rangle_2.code$,
 ("BR" label($\langle c \rangle_1.labin + 1$)),
 (label($\langle c \rangle_1.labin$) "NOP"),
 $\langle c \rangle_3.code$,
 (label($\langle c \rangle_1.labin + 1$) "NOP"))

Code Generation for Statements

```
<c>1 ::= while <be> do <c>2  
    <c>2.labin := <c>1.labin + 2  
    <c>1.labout := <c>2.labout  
    <c>1.code := concat(  
        ( label(<c>1.labin) "NOP" ),  
        <be>.code,  
        ( "BZ" label(<c>1.labin + 1) ),  
        <c>2.code,  
        ( "BR" label(<c>1.labin) )  
        ( label(<c>1.labin+1) "NOP" ) )
```

Code Generation for Statements

`<assign> ::= id := <ae>`

`<ae>.temp := 1`

`<assign>.code := concat(
 <ae>.code,
 ("STO" id.lexval))`

- For recursive languages, we need to access variables on the run-time call stack (multiple instances of `id.lexval`)

Code Generation for Expressions

```
<ae>1 ::= intconst
    <ae>1.code := < ("LOAD" intconst.lexval) >
    | id
    <ae>1.code := < ("LOAD" id.lexval) >
    | <ae>2 + <ae>3
    <ae>2.temp := <ae>1.temp
    <ae>3.temp := <ae>1.temp + 1
    <ae>1.code := concat(
        <ae>2.code,
        ( "STO" temp(<ae>1.temp) ),
        <ae>3.code,
        ( "ADD" temp(<ae>1.temp) ) )
```

Example for Code Generation

- Source program

if x = 42 then

 if a = b then

 y := 1

 else

 y := 2

else

 y := 3

Example for Code Generation

- Generated code for outer if statement

code for (x=42)

BZ L1

code for inner if

BR L2

L1: NOP

code for y := 3

L2: NOP

Inner If Statement

code for x = 42

BZ L1

code for a = b

BZ L3

code for y := 1

BR L4

L3: NOP

code for y := 2

L4: NOP

BR L2

L1: NOP

code for y := 3

L2: NOP

Code for Assignments

code for x = 42

BZ L1

code for a = b

BZ L3

LOAD 1

STO y

BR L4

L3: NOP

LOAD 2

STO y

L4: NOP

BR L2

L1: NOP

LOAD 3

STO y

L2: NOP

Summary: Attribute Grammars

- Useful for expressing arbitrary cycle-free walks over context-free parse trees
- Synthesized and inherited attributes
- Conditions to reject invalid parse trees
- Evaluation order depends on attribute dependencies

Summary: Attribute Grammars

- Realistic applications: for **type checking** and **code generation**
- “Global” data structures must be passed around as attributes
- Any data structure (sets, etc.) can be used as long as we work on paper
- The evaluation rules can call auxiliary functions
 - but the functions cannot have side effects